# Sales Effort Free Riding and Coordination: How Manufacturers can Help Brick-and-Mortar Stores Fight "Showrooming"? 

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#### Abstract

We study a supply chain with one manufacturer and two retail channels, where an online retailer offers a lower price and free-rides a brick-and-mortar retailer's sales effort. The free riding effect reduces brick-and-mortar retailer's desired effort level, and thus hurts the manufacturer's profit and the overall supply chain performance. To coordinate the efforts, we design two contracts: a selective rebate contract with price match and a revenue sharing contract with price match. For both contracts, our analysis goes with two cases: the online channel is owned by or independent of the manufacturer. The selective rebate contract coordinates the supply chain in both cases. It can also allocate the system profits arbitrarily between the supply chain players. Furthermore, in the case that the manufacturer owns the online channel, there exists a solution regime on the Pareto-optimal frontier in which both the manufacturer and the brick-and-mortar retailer's profits are improved from the baseline case. In addition, we show that the revenue sharing contract with price match is equivalent to the selective rebate contract.


## 1 Introduction

'Showrooming', or the so called sales effort free riding (see Shin 2007, Bernstein et al. 2009, Xing and Liu 2012), is an increasing problem for brick-and-mortar chains, at the same time that it's a boon for online stores. In early 2012, The Wall Street Journal (Zimmerman 2012) and The Time Magazine (Tuttle 2012) reported the tension between two retail giants, Amazon.com and Target, caused by showrooming. In December 2011, Amazon offered special discounts encouraging shoppers to use brick-and-mortar stores merely as showrooms, allowing consumers to scope out items in person before ultimately buying them at cheaper prices from Amazon.com (Indvik 2011). In January 2012, Target called to fight back. Manufacturers are being asked to
help Target figure out ways to solve the free riding problem (Zimmerman 2012). Lowering the wholesales prices Target pays to the manufacturers is the easiest way to accomplish that goal, however, a scenario manufacturers obviously won't like. The letter, signed by Target executives, reads: "What we aren't willing to do is let online-only retailers use our brick-and-mortar stores as a showroom for their products and undercut our prices without making investments, as we do, to proudly display your brands." Best Buy also offered price match to customers who find a cheaper price on their smartphones (Noguchi 2011).

Besides the competition from independent online stores, brick-and-mortar retailers also face challenges from the manufacturer owned direct channels. The rapid development of commerce on the Internet has made it easier for many manufacturers to engage in direct sales. According to the survey by Tedeschi (2000) in The New York Times, about $42 \%$ of top suppliers (e.g., IBM, Pioneer Electronics, Cisco System, Estee Lauder, and Nike) in a variety of industries had begun to sell directly to consumers over the Internet. While more and more manufacturers are engaging in direct retail channels, their brick-and-mortar retailer partners voice the belief that orders placed through a manufacturer's direct channel are orders that should have been placed through them. In a Fortune article (Brooker 1999), Home Depot was reported to issue a Godfather-esque directive to all suppliers selling products over the Internet, saying "We recognize that a vendor has the right to sell through whatever distribution channels it desires. However, we too have the right to be selective in regard to the vendors we select and we trust that you can understand that a company may be hesitant to do business with its competitors."

For suppliers, the threat from Home Depot is real because the brick-and-mortar retailers are indispensable. There's one aspect of the physical shopping experience that's still far superior to its online counterpart: Shopping in person is a physical experience because the consumer gets to touch the merchandise. No matter how many photos of an item on a website, no matter how extensive the description, and no matter how many customer reviews are there, buying online always comes with a higher degree of guesswork than buying in person. In addition to providing physical experience to customers, brick-and-mortar retailer can also stimulate demand by mailing advertisement posters, providing attractive shelf space, offering trial samples, and educating customers about the product with sales representatives.

For obvious reasons, both brick-and-mortar retailers and suppliers want to put an end to showrooming.

However, the tactics employed by both Target and Best Buy lack a cost sharing system between the suppliers and the retailers. Target asked the suppliers to cover the whole cost by reducing wholesale prices, while Best Buy swallowed the entire loss caused by reduced retail prices. To tackle sales effort free riding in a more balanced way, in this paper we design a selective rebate contract with price-match to help the manufacturer coordinate the supply chain. In this scheme, the brick-and-mortar retailer matches the online retailer's price if a customer shows the proof of the lower price. The manufacturer then offers a compensation rebate to the brick-and-mortar retailer for each sale under price match. Notice that the brick-and-mortar retailer would not offer the lower online price to all customers, since the manufacturer rebate compensates only part of the price difference. We examine two cases with/without the manufacturer owning the online channel. We show that when demand is influenced by sales effort, a properly designed selective rebate contract with price match can coordinate the supply chain, arbitrarily split the system profit and achieve Pareto optimality.

The selective rebate contract is different from the classical manufacturer rebate contracts (e.g. linear rebate and target rebate) in the sense that the rebates are only given for the sales with price match. The manufacturer leverages the selective rebate and the wholesale price to encourage the brick-and-mortar retailer to invest in sales effort. At the same time, the manufacturer avoids the sheer loss of profits caused by the massive reduction of marginal revenue as in the linear rebate.

In order to show that the selective rebate contract can be applied to a wide market context, we examine two supply chain structures. We first examine the supply chain structure in which the manufacturer owns the online retail channel. Such direct online channels are especially favored by industry leaders. For example, HP, the leading consumer PC manufacturer in the US, moved to online direct sales in the late 1990s and by $2004,26 \%$ of their orders were from the online direct sales (Burke 2004). Another telling example is Apple Inc., the world's number one consumer electronics vendor. In 2010, Apple online store sales grew by $90 \%$, thanks to the strong demand for iPad 2 and iPhone 4 s . We then analyze the supply chain structure in which the manufacturer does not own the online retailer. This supply chain structure is mostly applicable to the manufacturers not in a leadership position, such as the lesser known brands of ASUS in the computer market, and CREATIVE in the consumer electronics market. By investigating both supply chain structures, we show that the selective rebate contract is appealing to both the leading and non-leading manufacturers
who are battling the sales effort free riding conflict between retail channels.
We also extend our analysis to include revenue sharing contract. We show that revenue sharing and selective rebate contracts with price match are equivalent. For any selective rebate contract there exists a revenue sharing contract that generates the same cash flows between the manufacturer and the brick-andmortar retailer. However, if the administrative costs associated with monitoring revenues and collecting transfers are considered in implementing the revenue sharing contract, the manufacturer will prefer selective rebate contract over revenue sharing contract.

The rest of the paper is organized as follows. Section 2 provides a review of related literature. Section 3 presents the models of the selective rebate and revenue sharing contracts with price match, and the integrated supply chain when the manufacturer owns the online channel. We also analyze the equivalence between the two coordinating contracts. Section 4 shows that the selective rebate with price match contract still coordinates the supply chain in the second case where the manufacturer doesn't own the online retailer, as well as its equivalence to the revenue sharing contract. Finally, we conclude the paper in Section 5.

## 2 Literature review

This paper studies supply chain contract design to solve sales effort free riding between retail channels. In this section, we review the literature our research is related to: supply chain coordination, sales effort, free riding, channel rebate contracts, revenue sharing contract, and price match.

Supply chain coordination is a classic topic in the literature. Cachon (2003) analyzes every contract with their ability to coordinate the players' decisions across the supply chain. When coordination is achieved, the system profit of the supply chain is maximized. The question following supply chain coordination is the division of system profit. Some contracts have been proved to possess the ability of arbitrary division of system profit. For example, Cachon and Lariviere (2005) shows that the revenue sharing contract can achieve supply chain coordination while arbitrarily splitting the system profit between the manufacturer and the retailer. Cai (2010) studies channel coordination in a dual channel supply chain. He shows that revenue sharing contract can coordinate both the retailer-retailer and retailer-direct channel supply chains, but at different supply chain efficiencies. To our best knowledge, none of the previous literature has discussed
arbitrary profit division in a multichannel supply chain.
The impact of a retailer's sales effort on demand expansion has been broadly studied in the operations management literature (see Chu and Desai 1995, Lariviere and Padmanabhan 1997, Netessine and Rudi 2000, Taylor 2002, Gilbert and Cvsa 2003, Mukhopadhyay et al. 2008). Gurnani et al. (2007) investigate the impact of investment decisions (sales effort, price, and product quality etc.) on the supply chain profitability. Some researchers show that sales effort can also affect various properties of the supply chain, such as the demand uncertainty analysis by Heese and Swaminathan (2003) and risk aversion analysis by Suo et al. (2005). None of these papers consider contractual incentives to coordinate the supply chain.

Sales effort coordination has attracted significant attention from the supply chain management researchers. Cachon (2003) surveys the recent literature on sales effort and supply chain coordination. He shows that several supply chain contracts (e.g., sales rebate, buy back, or revenue sharing) can coordinate a supply chain with sales effort. He et al. (2009) examine a supply chain facing stochastic demand that depends on both sales effort and retail price. They find that only the returns policy can achieve supply chain coordination. Xie and Neyret (2009) show that the co-op advertising and pricing strategies can maximize the system profit in a one-manufacturer-and-one-retailer supply chain. Karray (2010) investigates the effects of horizontal joint promotions among retailers and show that this cost sharing strategy can improve each channel member's profit through demand expansion and higher margins in all the channels. Xing and Liu (2012) study sales effort coordination with stochastic demand. They show that the selective rebate contract with price match solves the free riding problem and coordinates the brick-and-mortar retailer's sales effort. However, none of the literature has achieved maximum system profit while investigating sales effort free riding and coordination.

There is a stream of literature extensively studying the free riding phenomenon in the fields of industrial organization and marketing (for a survey in a retail channel environment, see Carlton and Chevalier 2001 and Antia et al. 2004). For example, Carlton and Perloff (2004) shows that manufacturers can avoid free riding by using vertical restrictions. Studies on free riding in the field of operation management are rare. Bernstein et al. (2009) design a supply chain contract to increase channel competition to improve the manufacturer's profit. Shin (2007) shows that free riding may benefit both the free rider and the effort provider
because it softens price competition. Wu et al. (2004) examine a retail market where information service, a form of sales effort, is provided by retailers to help consumers identify their ideal products. Their analysis suggests that a retailer in this setting needs to develop the reputation for service provision to obtain positive profits. The retailer who chooses to free ride all the time loses the market share. Sigua and Chintagunta (2009) study a problem of sharing advertising cost among the franchisor and the franchisees, which resembles a supply chain with a manufacturer and multiple retailers. They show that franchisor's compensation to coordinate the franchisees' advertising efforts can maximize the supply chain system profit. However, none of the papers above consider sales effort free riding among asymmetric retailers (e.g., brick-and-mortar/onine retailers), as we do in the current paper.

Channel rebate, a broadly used incentive contract in the retail business, has been applied to coordinate the retailer's effort (see Lariviere 1998, Tsay et al. 1998, Cachon 2003, for surveys on supply chain contracts). Taylor (2002) shows that a target rebate and returns contract can coordinate retailer's sales effort. Krishnan et al. (2004) show that a buy back and manufacturer rebate contract can coordinate the supply chain, in which the retailer decides her order quantity first, then makes her effort decision after detecting a signal of the market demand. Taylor and Xiao (2009) compare rebate and returns contracts in a single channel supply chain with retailer's forecasting effort. They show that the manufacturer can achieve supply chain coordination with the optimal menu of returns contracts. However, none of the papers mentioned above has studied a manufacturer rebate based on the sales to a specific customer group and applied it in a multi-channel supply chain with sales effort.

There is a string of papers that investigate revenue sharing contracts. Dana and Spier (2001) study these contracts in the context of a perfectly competitive retail market. Pasternack (2005) studies a single retailer newsvendor model in which the retailer can purchase some units with revenue sharing and other units with a wholesale price contract. He does not consider supply chain coordination in his model. Cachon and Lariviere (2005) study revenue sharing contract in supply chain coordination. They find that revenue sharing contract alone cannot coordinate retail effort, so they develop a variation on revenue sharing, a quantity discount contract, for this setting. However, none of the papers have studied revenue sharing contract in the context of sales effort free riding.

The impact of price match has been examined in the economics and marketing literature. Hess and Gerstner (1991) show that price match helps to avoid price competition since the retailer becomes cautious to use price cut to compete with her price matching rivals. Chen and Narasimhan (2001) argue that price match generates not only a competition-alleviating effect, but also a competition-enhancing effect. The former case accords to Hess and Gerstner (1991). The latter effect comes from the fact that price match encourages consumers' price search behavior and thus exaggerates price competition. Corts (1996) studies the fluctuations in equilibrium prices caused by price match policy, and shows that price match facilitates customer segmentation according to the extent of the customers' information about rivals' prices. However, the previous studies mostly considered the price match policy as a marketing tactic, and focused on its impact on price competition. So far we have not noticed any literature that considers price match as a tactic to coordinate retailer effort. To the best of our knowledge, this is the first paper using price match for sales effort coordination with free riding between retailers.

## 3 Model with Online Channel Owned by the Manufacturer

### 3.1 Assumptions and notations

We assume the retail price is exogenous. Some people may argue that fixing retail price makes the demand model oversimplified. However, as recommended by Lariviere and Porteus (2001), "A fixed retail price keeps the underlying inventory problem sufficiently straightforward that one can study many aspects of supply chain interactions and incentives." In our case, the focus is sales effort and free riding, and Cachon and Lariviere (2001) also use fixed retail prices to analyze the demand forecast effort.

In this paper, we use an effort-dependant linear demand model, similar to Cachon and Lariviere (2005). They employ a linear deterministic model when analyzing retailer effort and revenue sharing contract. Such effort dependent demand model can also be found in Chu and Desai (1995) and Desiraju and Moorthy (1997).

The market demand is categorized into three customer groups: 1) The traditional customers who only shop in the brick-and-mortar stores. Denote their demand as $D_{b}=a_{b}+\tau_{b} \theta-p_{b}$, where $p_{b}$ is the brick-and-mortar retailer's retail price, and $\tau_{b}$ is the coefficient summarizing the demand boosting effect by sales
effort on the traditional consumers. Similarly, $\tau_{f}$ is for the free-riding consumers and $\tau_{o}$ is for the online only consumers; 2) The free-riding customers who take advantage of the brick-and-mortar retailer's sales effort but purchase online at a lower price. Define their demand as $D_{f}=a_{f}+\tau_{f} \theta-p_{o}$, where $p_{o}$ is the online retailer's retail price; 3) The online shoppers who only purchase through online stores, thus their demand is barely affected by the brick-and-mortar retailer's sales effort, defined as $D_{o}=a_{o}+\tau_{o} \theta-p_{o}$. Notice that, when $\theta=0$, the base demands are $a_{b}-p_{b}, a_{f}-p_{o}$, and $a_{o}-p_{o}$, respectively. Thus these three terms are assumed to be positive. The cost function of sales effort is $V(\theta)=h \theta^{2}$.

Assumption 1 The cost function of sales effort, $V(\theta)$, is an increasing convex function, and $V(0)=0$.

Taylor (2002) makes similar assumption on the cost function of sales effort. We assume the cost function is in the form of $V(\theta)=h \theta^{2}, h>0$. We name $h$ as the cost coefficient of effort, since $h$ clearly affects the costliness of effort and it is not the marginal cost.

Assumption 2 When the brick-and-mortar retailer's sales effort is 0, we have the following relationship for the base demand: $a_{b}-p_{b} \geq a_{f}-p_{o}$ and $a_{o}-p_{o} \geq a_{f}-p_{o}$.

Intuitively, under the base market demand, i.e., without the influence of sales effort, the number of the free riding customers should be no greater than the traditional shoppers and the online shoppers.

Assumption 3 Compared with the free-riding and online only customers, the traditional shoppers' demand is more sensitive to the brick-and-mortar retailer's sales effort, i.e., $\tau_{b}>\tau_{f}>\tau_{o}$.

This assumption is reasonable in that the traditional shoppers visit the brick-and-mortar retailer more frequently and are loyal to such retailers. On the other hand, the brick-and-mortar retailer understand the traditional shoppers better than free riding customers and tend to provide more effective sales effort to them, e.g., membership promotions based on customers' purchase history like Sam's club card, which free riding customers are hard to obtain because their purchases are realized online. This assumption is also consistent with Xing and Liu (2012).

### 3.2 The centralized supply chain

Before we investigate the selective rebate contract, we first look at the centralized supply chain, which focuses on supply chain coordination and thus serves as a benchmark to evaluate the efficiency of the decentralized system. The results in the centralized supply chain also facilitates the analysis of supply chain coordination under the selective rebate contract with price match in the decentralized system.

In the centralized system, the central planner only needs to decide the sales effort:

$$
\begin{equation*}
\Pi^{C}(\theta)=\left(p_{b}-c\right)\left(a_{b}+\tau_{b} \theta-p_{b}\right)+\left(p_{o}-c\right)\left(a_{o}+\left(\tau_{f}+\tau_{o}\right) \theta-2 p_{o}+a_{f}\right)-h \theta^{2} \tag{1}
\end{equation*}
$$

The first order and second order derivatives of $\Pi^{C}(\theta)$ are:

$$
\begin{gathered}
\frac{\partial \Pi^{C}(\theta)}{\partial \theta}=\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)-2 h \theta \\
\frac{\partial^{2} \Pi^{C}(\theta)}{\partial \theta^{2}}=-2 h<0
\end{gathered}
$$

Thus $\Pi^{C}(\theta)$ is concave in $\theta$ and we can obtain the optimal $\theta$ as follows:

$$
\begin{equation*}
\theta^{C *}=\frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)}{2 h} \tag{2}
\end{equation*}
$$

By substituting equation (2) into equation (1), $\Pi^{C *}$ can be obtained as follows:

$$
\begin{equation*}
\Pi^{C *}=\left(p_{b}-c\right)\left(a_{b}-p_{b}\right)+\left(p_{o}-c\right)\left(a_{o}-2 p_{o}+a_{f}\right)+\frac{\left(\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)\right)^{2}}{4 h} \tag{3}
\end{equation*}
$$

Equation (2) defines the optimal sales effort that maximizes the system profit to the level of Equation (3). Obviously, these two equations also represent the coordinated sales effort and system profit. They are the upper bounds of the optimal solutions of the decentralized models.

### 3.3 The selective rebate with price match contract

In this contract, the brick-and-mortar retailer matches the online channel's price if a customer shows the proof of the lower price. At the end of the selling season, the manufacturer who owns the online channel (the joint venture hereafter) offers a compensation rebate to the brick-and-mortar retailer for each sale under price match. The manufacturer decides the wholesale price $(w)$ and rebate $(u)$ to maximize his profit.

### 3.3.1 The brick-and-mortar retailer's profit

Because of the price match policy, the internet savvy customers purchase in the brick-and-mortar store. The brick-and-mortar retailer's profit is:

$$
\begin{equation*}
\Pi_{b}^{S}(\theta)=\left(p_{b}-w\right)\left(a_{b}+\tau_{b} \theta-p_{b}\right)+\left(p_{o}-w+u\right)\left(a_{f}+\tau_{f} \theta-p_{o}\right)-h \theta^{2} . \tag{4}
\end{equation*}
$$

The first order and second order derivatives of $\Pi_{b}^{S}(\theta)$ are:

$$
\begin{gathered}
\frac{\partial \Pi_{b}^{S}(\theta)}{\partial \theta}=\left(p_{b}-w\right) \tau_{b}+\left(p_{o}-w+u\right)\left(\tau_{f}+\tau_{o}\right)-2 h \theta \\
\frac{\partial^{2} \Pi_{b}^{S}(\theta)}{\partial \theta^{2}}=-2 h<0
\end{gathered}
$$

Thus $\Pi_{b}^{S}(\theta)$ is concave in $\theta$ and we can obtain the optimal $\theta$ as follows:

$$
\begin{equation*}
\theta^{*}=\frac{\left(p_{b}-w\right) \tau_{b}+\left(p_{o}-w+u\right)\left(\tau_{f}+\tau_{o}\right)}{2 h} \tag{5}
\end{equation*}
$$

### 3.3.2 The online retailer and manufacturer joint venture's profit

Since there is price match, the internet savvy customers purchase in the brick-and-mortar stores. The joint venture's profit becomes:

$$
\begin{equation*}
\Pi_{j}^{S}(w, u)=\left(p_{o}-w\right)\left(a_{o}-p_{o}\right)+(w-c-u)\left(a_{f}+\left(\tau_{f}+\tau_{o}\right) \theta-p_{o}\right)+(w-c)\left(a_{b}+\tau_{b} \theta-p_{b}+a_{o}-p_{o}\right) \tag{6}
\end{equation*}
$$

After substituting $\theta$ with equation (5), equation (6) can be rewritten as:

$$
\begin{align*}
\Pi_{j}^{S}(w, u) & =\left(p_{o}-w\right)\left(a_{o}-p_{o}\right)+(w-c-u)\left(a_{f}+\frac{\left(p_{b}-w\right) \tau_{b}\left(\tau_{f}+\tau_{o}\right)+\left(p_{o}-w+u\right) \tau_{f}^{2}}{2 h}-p_{o}\right)  \tag{7}\\
& +(w-c)\left(a_{b}+\frac{\left(p_{b}-w\right) \tau_{b}^{2}+\left(p_{o}-w+u\right) \tau_{b}\left(\tau_{f}+\tau_{o}\right)}{2 h}-p_{b}+a_{o}-p_{o}\right)
\end{align*}
$$

The system profit in the supply chain in terms of $w$ and $u$ is:

$$
\begin{align*}
\Pi_{j}^{S}(w, u)+\Pi_{b}^{S}(w, u)= & (w-c-u)\left(a_{f}+\frac{\left(p_{b}-w\right) \tau_{b}\left(\tau_{f}+\tau_{o}\right)+\left(p_{o}-w+u\right) \tau_{f}^{2}}{2 h}-p_{o}\right) \\
& +(w-c)\left(a_{b}+\frac{\left(p_{b}-w\right) \tau_{b}^{2}+\left(p_{o}-w+u\right) \tau_{b}\left(\tau_{f}+\tau_{o}\right)}{2 h}-p_{b}+a_{o}-p_{o}\right) \\
& +\left(p_{o}-w\right)\left(a_{o}-p_{o}\right)+\left(p_{b}-w\right)\left(a_{b}+\frac{\left(p_{b}-w\right) \tau_{b}^{2}+\left(p_{o}-w+u\right) \tau_{b}\left(\tau_{f}+\tau_{o}\right)}{2 h}-p_{b}\right)  \tag{8}\\
& +\left(p_{o}-w+u\right)\left(a_{f}+\frac{\left(p_{b}-w\right) \tau_{b} \tau_{f}+\left(p_{o}-w+u\right) \tau_{f}^{2}}{2 h}-p_{o}\right) \\
& -h\left(\frac{\left(p_{b}-w\right) \tau_{b}+\left(p_{o}-w+u\right) \tau_{f}}{2 h}\right)^{2}
\end{align*}
$$

Equation (8) doesn't reveal any information about the optimal decisions for the manufacturer, rather, it shows how the manufacturer's decisions affect the system profit. We will design a coordinating decision formula based on Equation (8) in the following section.

### 3.3.3 Supply chain coordination

In this section, we design the contract to achieve supply chain coordination without the consideration of voluntary compliance. The supply chain achieves the maximum system profit, same as the centralized supply chain, but the individual player's profit is not guaranteed to be improved. We will investigate the solution regime under Pareto improvement in a later section.

Here we introduce variable $\lambda$. Define $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}} \lambda$ and $w=\lambda+c$. Note that by such design, we actually construct $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}}(w-c)$.

Theorem 1 The selective rebate with price match contract achieves supply chain coordination. It obtains the same system profit as the centralized supply chain.

Proof By substituting $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}} \lambda$ and $w=\lambda+c$ into equation (8), we can rewrite (8) as follows:

$$
\begin{align*}
\Pi_{j}^{S}(\lambda)+\Pi_{b}^{S}(\lambda)= & -\frac{\tau_{b}}{\tau_{f}+\tau_{o}} \lambda\left(a_{f}+\frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}}{2 h} \tau_{f}-p_{o}\right) \\
& +\lambda\left(a_{b}+\frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}}{2 h} \tau_{b}-p_{b}+a_{o}-p_{o}\right)+\left(p_{o}-c-\lambda\right)\left(a_{o}-p_{o}\right) \\
& +\left(p_{b}-c-\lambda\right)\left(a_{b}+\frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}}{2 h} \tau_{b}-p_{b}\right)  \tag{9}\\
& +\left(p_{o}-c+\frac{\tau_{b}}{\tau_{f}} \lambda\right)\left(a_{f}+\frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}}{2 h} \tau_{f}-p_{o}\right)-\frac{\left(\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}\right)^{2}}{4 h} \\
= & \left(p_{b}-c\right)\left(a_{b}-p_{b}\right)+\left(p_{o}-c\right)\left(a_{o}-2 p_{o}+a_{f}\right)+\frac{\left(\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)\right)^{2}}{4 h}
\end{align*}
$$

Thus equation (9) equals (3).

We can transform other optimal results in terms of $\lambda$.
Equation (4) can be changed to:

$$
\begin{equation*}
\Pi_{b}^{S *}(\lambda)=\Pi^{C *}-\left(p_{o}-c\right)\left(a_{o}-p_{o}\right)-\lambda\left(a_{b}-p_{b}-a_{f}+p_{o}+\left(\tau_{b}-\tau_{f}\right) \frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)}{2 h}\right) \tag{10}
\end{equation*}
$$

where $\Pi^{C *}$ is the centralized system profit as equation (3).

Equation (6) can be changed to:

$$
\begin{equation*}
\Pi_{j}^{S *}(\lambda)=\left(p_{o}-c\right)\left(a_{o}-p_{o}\right)+\lambda\left(a_{b}-p_{b}+p_{o}-a_{f}+\left(\tau_{b}-\tau_{f}\right) \frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}}{2 h}\right) \tag{11}
\end{equation*}
$$

Theorem 1 indicates that there is a Pareto-optimal frontier in the optimal decision regime, determined by the intermediate decision variable $\lambda$ through $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}} \lambda$ and $w=\lambda+c$. On the Pareto-optimal frontier, the solutions are Pareto-optimal, which means we cannot find a solution to improve either player's profit without undermining the other's. Though the system profit has been maximized, the individual player's profit share is variable depending on the menu of the decisions, as shown in the following section.

### 3.3.4 Arbitrary split of the system profit

Arbitrary allocation of system profit is a property shared by several efficient supply chain contracts, e.g., buy back contract and revenue sharing contract. In this section, we show that the selective rebate with price match contract also possesses a similar property, though each player has a reserved level of profit share.

Theorem 2 Under selective rebate with price match contract, the system profit can be arbitrarily split by varying $\lambda$ among the supply chain players. Especially, the joint venture attains his highest profit when:

$$
\begin{equation*}
\lambda=\frac{\left(p_{b}-c\right)\left(a_{b}-p_{b}\right)+\left(p_{o}-c\right)\left(a_{f}-p_{o}\right)+\frac{\left(\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)\right)^{2}}{4 h}}{a_{b}-p_{b}+p_{o}-a_{f}+\left(\tau_{b}-\tau_{f}\right) \frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)}{2 h}} \tag{12}
\end{equation*}
$$

Proof From assumption $a_{b}-p_{b}>a_{f}-p_{o}$, we know that $a_{b}-p_{b}-a_{f}+p_{o}>0$, thus in equation (10), the brick-and-mortar retailer's profit is a decreasing function of $\lambda$. Obviously, the joint venture's profit is increasing with $\lambda$ as shown in (11). Equation (12) is obtained by equating (10) to zero. Notice that the nominator and denominator of equation (12) are both positive, thus there always exists a value of $\lambda$ that makes the brick-and-mortar retailer obtain zero share of the system profit.

### 3.3.5 Pareto improvement

Though we have shown that the selective rebate contract can achieve arbitrary profit division and Pareto optimality, it is interesting to find out under what condition the selective rebate contract can improve the profitability for both the manufacturer joint venture and the brick-and-mortar retailer.

Naturally, after shifting the free-riding customers' demand from the online channel to the brick-andmortar retailer, the brick-and-mortar retailer's profit will be higher under the selective rebate with price match. The intriguing question is, under what condition, the joint venture will be better off.

Theorem 3 The joint venture's profitability is affected by the cost coefficient of sales effort $h$,

- When $h \geq \frac{\tau_{f}\left(c \tau_{b}-p_{b} \tau_{b}+c \tau_{f}-p_{o} \tau_{f}\right)}{-2\left(a_{f}-p_{o}\right)}$, the joint venture is not worse off in the selective rebate than in the baseline;
- When $h<\frac{\tau_{f}\left(c \tau_{b}-p_{b} \tau_{b}+c \tau_{f}-p_{o} \tau_{f}\right)}{-2\left(a_{f}-p_{o}\right)}$, the optimal wholesale price $w_{x}=p_{b}-\frac{2 h\left(\tau_{b} \tau_{f}\right)}{a_{f}-p_{o}}$;
- When $p_{b}-p_{o} \leq \frac{2 h \tau_{b} \tau_{f}}{a_{f}-p_{o}}, w_{x} \leq p_{o}$.

Proof The difference between the joint venture's profit under the selective rebate and the baseline case is:

$$
\begin{align*}
\Pi_{d i f f}= & -\left(a_{o}-p_{o}\right)\left(-w+p_{o}\right)+\left(-w+p_{o}\right)\left(a_{f}+a_{o}-2 p_{o}-\frac{\left(w-p_{b}\right) \tau_{b} \tau_{f}}{2 h}\right) \\
& +(-c+w)\left(a_{b}+a_{f}+a_{o}-p_{b}-2 p_{o}-\frac{\left(w-p_{b}\right) \tau_{b}^{2}}{2 h}-\frac{\left(w-p_{b}\right) \tau_{b} \tau_{f}}{2 h}\right) \\
& -(-c+w)\left(a_{b}+a_{o}-p_{b}-p_{o}+\frac{\tau_{b}\left(-w \tau_{b}+p_{b} \tau_{b}-w \tau_{f}+p_{o} \tau_{f}+(-c+w)\left(\tau_{b}+\tau_{f}\right)\right)}{2 h}\right)  \tag{13}\\
& -\left(-c+w-\frac{(-c+w)\left(\tau_{b}+\tau_{f}\right)}{\tau_{f}}\right)\left(a_{f}-p_{o}+\frac{\tau_{f}\left(-w \tau_{b}+p_{b} \tau_{b}-w \tau_{f}+p_{o} \tau_{f}+(-c+w)\left(\tau_{b}+\tau_{f}\right)\right)}{2 h}\right)
\end{align*}
$$

Let Equation 13 equate 0, we have two solutions for $w^{*}$, $\frac{c \tau_{b}+c \tau_{f}-p_{o} \tau_{f}}{\tau_{b}}$ and $\frac{2 h a_{f}-2 h p_{o}+p_{b} \tau_{b} \tau_{f}}{\tau_{b} \tau_{f}}$. The larger one $\frac{2 h a_{f}-2 h p_{o}+p_{b} \tau_{b} \tau_{f}}{\tau_{b} \tau_{f}}$ is defined as $w_{x}$.

So $w_{x}$ is always smaller than $p_{b}$. Larger $h$ leads to smaller $w_{x}$. Also smaller $a_{f}-p_{o}$ or larger $\tau_{b}, \tau_{f}$, result in smaller $w_{x}$.

Theorem 3 shows that when $p_{b}-p_{o} \leq \frac{2 h \tau_{b} \tau_{f}}{a_{f}-p_{o}}$, there always exists a solution regime that guarantees at least one of the player's profit is improved from the baseline case, and none of the players is worse off. Intuitively, the wider the gap of $p_{b}-p_{o}$, the harder it is to find Pareto-improving solutions. This can be easily understood as follows: in order to stimulate the brick-and-mortar retailer, the manufacturer needs to provide partial compensation for the price difference $p_{b}-p_{o}$. The larger $p_{b}-p_{o}$ implies a larger profit transfer from the manufacturer to the brick-and-mortar retailer, thus it is harder to find a solution that doesn't undermine the manufacturer's profit share.

The following numerical analysis aims to further elaborate Theorem 3.
The standard setting for the parameters in the numerical analysis is as follows: $c=2, a_{b}=30, a_{f}=$ $11, a_{o}=26, p_{b}=10, p_{o}=8, \tau_{b}=3, \tau_{f}=1$.

Among the parameters that affect the Pareto improvement of the supply chain players' profitability, the cost coefficient of sales effort $(h)$ dominates the trend.

Figure 1 and Figure 2 show the different Pareto improvement scenarios when $h$ is increased from 1 to 2 .


Figure 1: The profit share with $h=1$.


Figure 2: The profit share with $h=2$.

In Figure 1, the joint venture's profit share is not always higher in the selective rebate than in the baseline case. The Pareto-improving solution regime is within the two intersection points of $w$. The right intersection is $w_{x}$. When $h$ is increased to 2 in Figure 2, we can see the joint venture's profit share in the rebate case is always higher than in the baseline case.

### 3.4 Revenue Sharing Contract

Under a revenue-sharing contract, a retailer pays a supplier a wholesale price for each unit purchased, plus a percentage of the revenue the retailer generates. Such contracts have become more prevalent in the videocassette rental industry relative to the more conventional wholesale price contract.

### 3.4.1 The model

In a supply chain consisted of one brick-and-mortar retailer and one manufacturer owning an online retail channel, transactions between the retailer and manufacturer are governed by a revenue sharing contract. This contract contains two decision terms, $r$ and $w_{r} . r$ is the share of retail revenue the manufacturer receives, i.e., given retail revenues $\Pi_{b}$, the retailer must transfer $r \Pi_{b}$ to the manufacturer but retains the remaining $(1-r) \Pi_{b}$. It is natural to assume $r \in[0,1]$, even though that restriction is not strictly required. We do not include in our model the administrative costs associated with monitoring revenues and collecting transfers. In other words, we assume the cost of implementation has no impact on the contract the supplier oers or the quantity the retailer purchases. (Implementation costs, of course, may impact whether revenue sharing is adopted at all.) $w_{r}$ is the wholesale price. Note that a standard wholesale-price contract is a revenue-sharing contract with $r=0$.

The brick-and-mortar retailer's profit is:

$$
\begin{equation*}
\Pi_{b}(\theta)=\left((1-r) p_{b}-w_{r}\right)\left(a_{b}+\tau_{b} \theta-p_{b}\right)+\left((1-r) p_{o}-w_{r}\right)\left(a_{f}+\tau_{f} \theta-p_{o}\right)-h \theta^{2} \tag{14}
\end{equation*}
$$

The approach to obtain the optimal decisions based on the first and second order derivatives is the same as in Section 3.4, thus we omit it.

$$
\begin{equation*}
\theta^{*}\left(w_{r}, r\right)=\frac{-w_{r} \tau_{b}+p_{b} \tau_{b}-u p_{b} \tau_{b}-w_{r} \tau_{f}+p_{o} \tau_{f}-r p_{o} \tau_{f}}{2 h} \tag{15}
\end{equation*}
$$

The joint venture's profit is:

$$
\begin{equation*}
\Pi_{j}\left(w_{r}, r\right)=\left(w_{r}-c\right)\left(a_{o}+\tau_{o} \theta-p_{o}\right)+\left(w_{r}-c+r p_{o}\right)\left(a_{f}+\tau_{f} \theta-p_{o}\right)+\left(w_{r}-c+r p_{b}\right)\left(a_{b}+\tau_{b} \theta-p_{b}\right) \tag{16}
\end{equation*}
$$

### 3.4.2 Equivalence to the selective rebate contract

In this section, we show that revenue sharing and selective rebate contracts are equivalent. For any selective contract there exists a revenue sharing contract that generates the same cash flows between the manufacturer and the brick-and-mortar retailer.

In the selective rebate contract, the brick-and-mortar retailer pays $w_{s}-u\left(w_{s}\right.$ is the wholesale price in the selective rebate contract) for each unit sold to the free-riding customers under price match, and $w_{s}$ for each unit sold to the traditional customers. In the revenue sharing contract, the brick-and-mortar retailer pays $w_{r}+r p_{o}$ ( $w_{r}$ is the wholesale price in the revenue sharing contract) for each unit sold to the free-riding customers under price match, and $w_{r}+r p_{b}$ for each unit sold to the traditional customers.

To get $w_{s}-u=w_{r}+r p_{o}$ and $w_{r}+r p_{b}=w_{s}$, we introduce $r=\frac{\left(\tau_{b}+\tau_{f}+\tau_{o}\right)}{\tau_{f}\left(p_{b}-p_{o}\right)} \lambda$ and $w_{r}=\lambda+c-\frac{\left(\tau_{b}+\tau_{f}+\tau_{o}\right) p_{b}}{\tau_{f}\left(p_{b}-p_{o}\right)} \lambda$ into $w_{r}+r p_{o}$ and $w_{r}+r p_{b}$, and $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}} \lambda$ and $w_{s}=\lambda+c$ into $w_{s}-u$ and $w_{s}$, thus the two contracts result in the same profits for the retailer and the manufacturer for any combination of free riding and traditional customers'demand.

By now we have shown that the selective rebate and revenue sharing contracts with price match are equivalent. Therefore we can obtain the following theorem.

Theorem 4 The revenue sharing contract coordinates the supply chain with $r=\frac{\left(\tau_{b}+\tau_{f}+\tau_{o}\right)}{\tau_{f}\left(p_{b}-p_{o}\right)} \lambda$ and $w_{r}=$ $\lambda+c-\frac{\left(\tau_{b}+\tau_{f}+\tau_{o}\right) p_{b}}{\tau_{f}\left(p_{b}-p_{o}\right)} \lambda$.

Notice that $w-c=\lambda\left(1-\frac{\left(\tau_{b}+\tau_{f}\right) p_{b}}{\tau_{f}\left(p_{b}-p_{o}\right)}\right)$ and $1-\frac{\left(\tau_{b}+\tau_{f}\right) p_{b}}{\tau_{f}\left(p_{b}-p_{o}\right)}<0$. Considering the revenue sharing rate $r \geq 0$, we observe that $w \leq c$, which means that in the coordinating revenue sharing contract, the wholesale price should not be higher than the production cost. The manufacturer shares the cost of sales effort with the brick-and-mortar retailer. The manufacturer's revenue comes from the brick-and-mortar retailer's sales revenue, and thus is directly affected by the retailer's sales effort. In addition, the manufacturer shares the risk of demand uncertainty with the brick-and-mortar retailer.

## 4 Extension: Independent Online Channel

In this section, we discuss the supply chain with an independent online retailer. The manufacturer sells products through both retail channels with the same wholesale price. The reason to extend the previous model to include an independent online retail is twofold: 1) practically, many manufacturers sell their products through independent online retails, such as Amazon.com. By analyzing the behavior of the selective rebate contract in this extended scenario, we can increase the practical value of the selective rebate since it can be applied in a wider market context; 2) theoretically, no contracts have been revealed to coordinate a supply chain with channel conflicts on sales effort free riding. In this section, we show that the selective rebate contract is the first contract to achieve the aforementioned property.

### 4.1 The selective rebate with price match contract

### 4.1.1 The model

The manufacturer decides wholesale price $(w)$ and target rebate $(u)$ to maximize his profit. The brick-andmortar retailer's profit is identical to that in $\S 3.3$ thus omitted.

Since there is price match, the internet savvy customers purchase in the brick-and-mortar stores. The online retailer's profit becomes:

$$
\begin{equation*}
\Pi_{o}^{S}=\left(p_{o}-w\right)\left(a_{o}++\tau_{o} \theta-p_{o}\right) \tag{17}
\end{equation*}
$$

The manufacturer's decisions are the wholesale price and the rebate. His profit is:

$$
\begin{equation*}
\Pi_{m}^{S}(w, u)=(w-c-u)\left(a_{f}+\tau_{f} \theta-p_{o}\right)+(w-c)\left(a_{b}+\tau_{b} \theta-p_{b}+a_{o}-p_{o}\right) \tag{18}
\end{equation*}
$$

### 4.1.2 Supply chain coordination

In the same spirit of $\S 3.3 .3$, we hereby introduce variable $\lambda$. Define $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}} \lambda$ and $w=\lambda+c$. Note that by such design, we actually construct $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}}(w-c)$.

Theorem 5 The selective rebate with price match contract achieves supply chain coordination. It obtains the same system profit as the centralized supply chain.

Proof The proof is similar to that of Theorem 1, thus omitted.

The brick-and-mortar retailer's profit as a function of $\lambda$ is:

$$
\begin{equation*}
\Pi_{b}^{S *}(\lambda)=\Pi^{C *}-\left(p_{o}-c\right)\left(a_{o}-p_{o}\right)-\lambda\left(a_{b}-p_{b}-a_{f}+p_{o}+\left(\tau_{b}-\left(\tau_{f}+\tau_{o}\right)\right) \frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)}{2 h}\right), \tag{19}
\end{equation*}
$$

where $\Pi^{C *}$ is the centralized system profit as equation (3).
The online retailer's profit is:

$$
\begin{equation*}
\Pi_{o}^{S *}(\lambda)=\left(p_{o}-c\right)\left(a_{o}-p_{o}\right)-\lambda\left(a_{o}-p_{o}\right) . \tag{20}
\end{equation*}
$$

The manufacturer's profit is:

$$
\begin{equation*}
\Pi_{m}^{S *}(\lambda)=\lambda\left(a_{b}-p_{b}+a_{o}-a_{f}+\left(\tau_{b}-\left(\tau_{f}+\tau_{o}\right)\right) \frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right)\left(\tau_{f}+\tau_{o}\right)}{2 h}\right) . \tag{21}
\end{equation*}
$$

### 4.1.3 Division of the system profit

Theorem 6 Under selective rebate with price match contract, the system profit can be arbitrarily split by varying $\lambda$ among the supply chain players. Especially, when $\lambda=p_{o}-c$, the manufacturer attains his highest profit.

Proof From assumption $a_{b}-p_{b}>a_{f}-p_{o}$, we know that $a_{b}-p_{b}-a_{f}+p_{o}>0$, thus in equation (19), the brick-and-mortar retailer's profit is a decreasing function of $\lambda$. Obviously, the online retailer's profit is also decreasing with $\lambda$ as shown in (20). Since we assume $a_{o}-p_{o} \geq a_{f}-p_{o}$, thus $a_{o}>a_{f}$, then the manufacturer's profit is increasing in $\lambda$ as shown in (21).

In the selective rebate with price match contract, by increasing $\lambda$, the manufacturer increases $w$ and $u$ altogether and obtains a higher profit, based on equation (21).

Let's consider two borderline cases: $\lambda=0$ and $\lambda=p_{o}-c$. When $\lambda=0, u=0$ and $w=c$, then we have $\Pi_{b}^{S *}=\Pi^{C *}-\left(p_{o}-c\right)\left(a_{o}-p_{o}\right), \Pi_{o}^{S *}=\left(p_{o}-c\right)\left(a_{o}-p_{o}\right), \Pi_{m}^{S *}=0$. In this case, manufacturer sells at the marginal cost, eliminating double marginalization, and naturally the supply chain coordinates. But the manufacturer's share of system profit is 0 . When $\lambda=p_{o}-c, u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}}\left(p_{o}-c\right)$ and $w=p_{o}$, then we have $\Pi_{b}^{S *}=\Pi^{C *}-\left(p_{o}-c\right)\left(a_{b}-p_{b}+a_{o}-a_{f}+\left(\tau_{b}-\tau_{f}\right) \frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}}{2 h}\right), \Pi_{o}^{S *}=0, \Pi_{m}^{S *}=\left(p_{o}-c\right)\left(a_{b}-p_{b}+a_{o}-a_{f}\right)$.

We can see that the division of system profit is not as flexible as from 0 to $100 \%$. Though the manufacturer and the online retailer could get a zero share of system profit (but not at the same time), the brick-andmortar retailer is reserved for a minimum profit as $\Pi^{C *}-\left(p_{o}-c\right)\left(a_{b}-p_{b}+a_{o}-a_{f}+\left(\tau_{b}-\tau_{f}\right) \frac{\left(p_{b}-c\right) \tau_{b}+\left(p_{o}-c\right) \tau_{f}}{2 h}\right)$, thanks to the sales to the traditional shoppers who always bring net profit to the brick-and-mortar retailer, due to $p_{b}>p_{o} \geq w$.

### 4.2 Revenue Sharing Contract

This section considers a revenue sharing contract with an independent online retailer. The online retailer loses her free riding customers to the brick-and-mortar stores due to price match. The manufacturer doesn't directly compensate the online stores for her demand drain, but by coordinating wholesales price and sales effort to achieve so.

### 4.2.1 The model

The revenue sharing contract with price match contains two decision terms, $r$ and $w_{r} . r$ is the share of retail revenue the manufacturer receives, and $w_{r}$ is the wholesale price.

The brick-and-mortar retailer's profit is identical to that in $\S 3.4$, thus omitted.
The online retailer's profit is:

$$
\begin{equation*}
\Pi_{o}=\left(p_{o}-w_{r}\right)\left(a_{o}++\tau_{o} \theta-p_{o}\right) . \tag{22}
\end{equation*}
$$

The manufacturer's profit is:

$$
\begin{equation*}
\Pi_{m}\left(w_{r}, r\right)=\left(w_{r}-c+r p_{o}\right)\left(a_{f}+\tau_{f} \theta-p_{o}\right)+\left(w_{r}-c+r p_{b}\right)\left(a_{b}+\tau_{b} \theta-p_{b}\right)+w_{r}\left(a_{o}++\tau_{o} \theta-p_{o}\right) \tag{23}
\end{equation*}
$$

### 4.2.2 Equivalence to the selective rebate contract

To get $w_{s}-u=w_{r}+r p_{o}$ and $w_{r}+r p_{b}=w_{s}$, we introduce $r=\frac{\left(\tau_{b}+\tau_{f}+\tau_{o}\right)}{\tau_{f}\left(p_{b}-p_{o}\right)} \lambda$ and $w_{r}=\lambda+c-\frac{\left(\tau_{b}+\tau_{f}+\tau_{o}\right) p_{b}}{\tau_{f}\left(p_{b}-p_{o}\right)} \lambda$ into $w_{r}+r p_{o}$ and $w_{r}+r p_{b}$, and $u=\frac{\tau_{b}+\tau_{f}+\tau_{o}}{\tau_{f}+\tau_{o}} \lambda$ and $w_{s}=\lambda+c$ into $w_{s}-u$ and $w_{s}$, thus the two contracts result in the same profits for the retailer and the manufacturer for any combination of free riding and traditional customers'demand.

By now, we have shown that the revenue sharing contract is equivalent to the selective rebate contract when the online channel is independent of the manufacturer. However, such equivalence only effects the coordinated sales effort and the system profit. The system profit is not arbitrarily divisible to the extent that a Pareto-improving solution can be found for all the three supply chain players as discussed in $\S 4.4$.

## 5 Conclusion

This paper examines the effectiveness of selective rebate contract with price match in coordinating a supply chain with retail channel conflicts caused by sales effort free riding. The price match policy diverts the demand of the internet savvy customers from the online channel to the brick-and-mortar retailer. By doing so, the brick-and-mortar retailer's sales effort is rewarded with the internet savvy customer's demand. In addition, the manufacturer provides partial compensation to the brick-and-mortar retailer to offset her loss due to price match. Such selective rebate boosts the brick-and-mortar retailer's order quantity and sales effort, achieves supply chain coordination and maximum system profits, and thus increase the supply chain efficiency.

This paper also demonstrates the superiority of the selective rebate over a traditional linear rebate in achieving supply chain coordination in a way that is attractive to the supply chain players involved. Under a linear rebate, the manufacturer induces the retailer to exert additional effort and order a larger quantity by increasing the retailer's marginal revenue. However, the manufacturer fully bears the financial burden of increasing the retailer's marginal revenue. A selective rebate offers an advantage to the manufacturer. By setting the partial rebate value, the manufacturer can induce the retailer to behave in a way that reflects the marginal revenue of the rebate while shielding the manufacturer from the full cost of doing so.

In addition, revenue sharing contract with price match is also studied in this paper. The deep root of equivalence between the selective rebate and revenue sharing contract with price match explains why both contracts can coordinate the supply chain with the manufacturer owning the online channel, and arbitrarily split the system profit. For any selective contract there exists a revenue sharing contract that generates the same cash flows. However, if the administrative costs associated with monitoring revenues and collecting transfers are considered in implementing the revenue sharing contract, the manufacturer will prefer selective
rebate contract over revenue sharing contract.
There are several areas that can be further explored with the selective rebate contract. For example, we have taken the retail prices to be exogenous. Exploring the manufacturer's use of rebates as an instrument that influences the retailer's pricing decision (e.g., to stimulate demand by driving down retail prices) may be a promising area for research. Finally, our analysis suggests that price matching, which is commonly used by retailers for price competition, can be used productively in a supplier-retailer rebate contract to mitigate channel conflicts. This suggests that it may be fruitful for researchers to apply the selective rebate contract in other channel conflicts beyond sales effort free riding.

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